Determination of the Internal Friction Angle of Gravel based on the p-q Graph for Direct Shear Tests

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ABSTRACT

Since its simplicity and availability, direct shear test is a current one. This test, compared to the triaxial test, provides different values for the parameters of the gravel due to the specific shape of the test device and the test approach. Triaxial testing, due to the feasibility of providing conditions similar to the natural behavior of gravel in a natural state, can provide more realistic values of material strength parameters. Due to being time consuming, costliness, problems in performing a triaxial test and the difficulty of sample preparation, direct shear test is still used more than triaxial test. In this paper, by studying the results of direct shear test and triangular experiments on gravel material, the stress paths in p - q space and the internal friction angle of these two tests are compared and the relation between them is obtained. To ensure the obtained results, large-scale triaxial and direct shear tests are used. The values of φ, τ_f and σ_n are calculated in every direct shear test, and using different lateral pressure coefficients (k), the failure points in the space p and q are obtained. Using the points, failure envelope is depicted in these graphs and then, using the relationships, the gradient of the failure envelope is converted to the friction angle of the sample; and, it is compared with the triaxial test results. Subsequently, the stress path is studied via the results, and it is compared with the stress path of the triaxial test. Finally, it can be concluded that, when the internal friction angle of the gravel in direct shear test is analyzed via lateral pressure coefficient (kp), it is approximately the same with the internal friction angle obtained by the triaxial test in p- q space.

Keywords: *Internal friction angle, p-q space, Direct shear test, Triaxial test.*

1. Introduction

In recent decades, the extensive use of gravel in geotechnical engineering, such as the construction of earth dam, road pavement and dock construction, has necessitated a thorough understanding of the behavior of such material. Among various applications of this material, dam construction is the most notable one [1]. One of the factors that makes it possible to know more about this material is its stress path graph and investigation of failure envelope including the internal friction angle (φ) in p-q graphs. In laboratory or site, The behavior of soil or depends on the history of applied loads and strains. In other words, the behavior of the soil depends not only on the initial and ultimate stresses, but also on how the stress and strain change previously.The stress path shows the variations of stress in the soil sample. The stress state is usually expressed via p and q space [2]. Stress paths were first introduced by Lamb. Subsequently, Wood developed various geotechnical issues in this field, then it was investigated in detail by Whitman and Lambe [3]. According to review studies, previous researches were conducted via stress path of triaxial tests and stress path of triaxial test was not considered.

Therefore, one of the aims of this research is investigating the stress path of the direct shear test for dry gravel. Sadrnejad did a detailed descriptive study of the total stress path via triaxial test; moreover, the gradient of failure envelope in p-q space is 1/3 [2]. The aim of this study is to investigate the stress path of large-scale direct shear test and compare it with graphs of triaxial tests. Then, the failure envelope is drawn in p-q space and the internal friction angle (φ) of soil is studied with using equations; moreover, the result is compared with the triaxial test. Since the internal friction angle is different in direct shear and triaxial tests [4], it is, therefore, sought to find a lateral pressure coefficient (k) to obtain the nearest internal friction angle of direct shear test to the internal friction angle of triaxial test. To find the hypothesis, the stress path in the p-q space of the sample of the direct shear test is drawn for different lateral pressure coefficients (k), ie. at rest (k_0), active (k_a) and passive (k_p) cases, and the combinations of these states such as (ka + kp)/2 and (k_0 + k_a + k_p)/3. Furthermore, it is investigated that which of these coefficients causes the internal friction angle of the direct shear test to be nearer to the internal friction angle of the triaxial test. To design a structure consisting of gravel, the design parameters of the materials must be properly recognized. In this regard, using large-scale laboratory tests is very useful as large dimensions of

these materials has created difficulties in performing laboratory tests and adapting their results to practical conditions [2]. To ensure accurate results, the data of a large-scale test has been used. The current dimensions of a large-scale direct shear test are 30x30 cm and 50x50 cm. By comparing the results of direct shear tests on 30×30 cm and 50×50 cm samples of gravel, Reyhani stated that, this material has relatively similar behavior in both tests, and no significant variations have been observed in shear strength parameters [5]. In this paper, the results of $30 \times 30 \times 15$ cm sample are used. The lateral pressure in the direct shear test is also considered to be $k\sigma_n$ where k and σ_n are the lateral pressure coefficient and the normal stress of direct shear test respectively.

2-Methodology

Given the fact that, the friction angle is different in triaxial and direct shear test, we consider different lateral pressure coefficients, and seek the lateral pressure coefficient, so that it can be used to convert the internal friction angle of direct shear test to the internal friction angle of triaxial test. In the study, the lateral pressure coefficients are calculated via [6]:

$$k_a=\tan^2(45-\varphi/2) \tag{1}$$
$$k_0=1-\sin\varphi \tag{2}$$
$$k_p=\tan^2(45+\varphi/2) \tag{3}$$

Where, k_0 is the lateral pressure coefficient of the at rest state, k_a is the lateral pressure coefficient of the active state, k_p is the lateral pressure coefficient of the passive state and φ is the internal friction angle of the material. In the results of this study, every large-scale test is done with different normal stresses (σ_n) of 111,222,444,666 and 777 kPa, and for each normal stress, the maximum shear stress (τf) is found separately. According to the results of the large-scale tests, φ, τ_f and σ_n are calculated. Furthermore, by programming in Excel software, σ_1 and σ_3 are obtained, then p and q are calculated respectively, and a failure envelope is depicted in p-q. First, the different lateral pressure coefficients, i.e. at rest (k_0), active (k_a), passive (k_p) and their combinations such as (k_a+k_p)/2 and ($k_a+k_p+k_0$)/3, are calculated. And, failure envelope is drawn in p-q graph.

In this regard, k is multiplied in σ_n to calculate lateral pressure. Subsequently, by depicting σ_n, the lateral pressure (k σ_n) and τ_f (maximum shear stress) in the Mohr circle and performing geometric calculations, principal stresses (σ_1 and b3) can be obtained. The differential stress (q) and mean stress (p) are calculated via the equations 4 and 5:

$$p = (\sigma_3 + \sigma_1)/2 \qquad\qquad (4)$$
$$q = (\sigma_1 - \sigma_3)/2 \qquad\qquad (5)$$

Where, σ_3 and σ_1 are the maximum and minimum principal stress at Mohr circle respectively. and 1b of the original stress at least in the Moore circle. Performing the previous steps, 5 points are shown in the p-q graphs, which they had passed through a line, then its equation is found; therefore, five failure envelope with different lateral pressure coefficient (k) are drawn in p-q graphs. It should be noted that, the written equations in Excel are based on Mohr-Coulomb's formulas. Das mentioned that the gradient of failure envelope (tan α) is equal to sin φ in the p-q graph, where φ is the internal friction angle of soil in the triaxial test. Because the results of the graphs are obtained from the direct shear test, we have φ_{ps}. Therefore, it can be converted to φ_{tr} using the relation [7]:

$$\varphi_{ps} = 1.15\varphi_{tr} \qquad\qquad (6)$$

Where, φ_{ps} and φ_{tr} are internal friction angle of soil in direct shear and triaxial tests respectively.

Consequently, the φ_{tr} obtained from the large-scale triaxial test is required to investigate that which lateral pressure coefficient makes the internal friction angle of the material in the direct shear test to be the nearest to the internal friction angle in the triaxial test. In order to obtain φ_{tr}, the gradient of the maximum shear strength vs. vertical surcharge graph, which is obtained from a large-scale triaxial test, is calculated. By using the Plot Digitizer software, three distinct points of the graph are defined, whose coordinates are shown in the graph. Then their coordinates are entered, and the software is calibrated. It can be used to find the coordinates of any point.

Because the gradient of the line is equal to tan φ, the points on the failure envelope are determined and the gradient is calculated via their coordinates, then by calculating $\tan^{-1}$ of the result, φtr is calculated. Finally, the internal friction angle of the soil in the direct shear and the triaxial tests are compared.

Subsequently, the stress paths of the large-scale direct shear test are investigated. In order to draw stress path, some points before the failure of sample must be analyzed. Since σ_n is constant in every test, points with equal σ_n and τ less than failure shear are studied, and they are converted to points in p-q graph via Excel software. It was observed that, increasing τ makes p to be constant, and increases q; therefore, in vertical line, the stresses increase up to the failure envelope or the sample is failed at this point.

3-Results

The below tables and figures are the results of this research. In tables 1 to 5, the normal stress and shear stress are constant and k is different. In Figures 1 to 5, the gradient of failure envelope is assumed to be tan(x).

Table 1: The values of p and q (kPa) with k0 as the lateral pressure coefficient

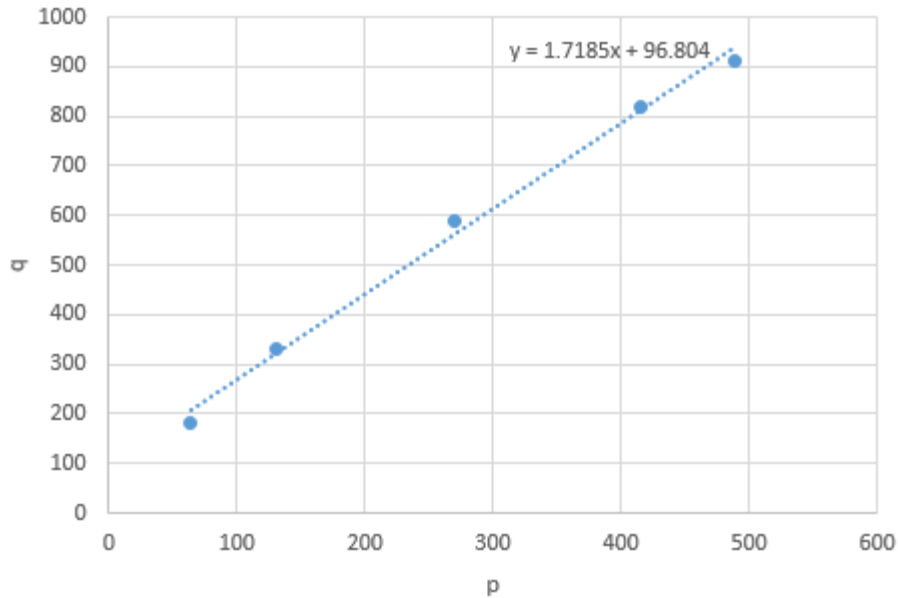| σ_n	τp	k0	k0σ_n		σ_1	σ_3		p	q
			0	0	0	0	0	0	0
111	176	0.152	16.872	47.064	182.184	246.12	-118.248	63.936	182.184
222	318.3	0.181	40.182	90.909	331.0277	462.1187	-199.937	131.091	331.0277
444	562.3	0.212	94.128	174.936	588.8836	857.9476	-319.82	269.064	588.8836
666	779	0.245	163.17	251.415	818.5661	1233.151	-403.981	414.585	818.5661
777	865.7	0.257	199.689	288.6555	912.556	1400.901	-424.212	488.3445	912.556

Fig 1. p - q graph of failure envelope (via k_0).

According to Table 1 and Figure 1, the gradient of failure envelope is 1.7185. Considering the equation of $\tan(x) = \sin\varphi$ and the gradient of graph which is greater than 1, k_0 could not be used for analyzing the internal friction angle of direct shear test and converting it to the internal friction angle of triaxial test.

Table 2: The values of p and q (kPa) with ka as the lateral pressure coefficient

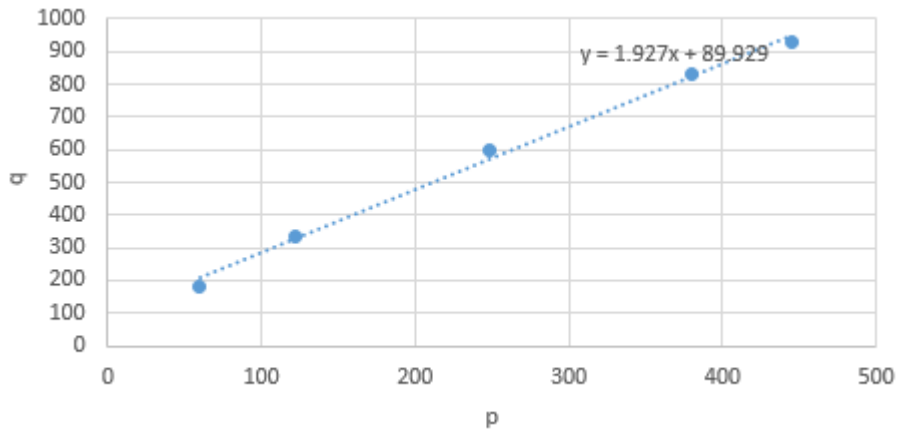| бn	τр	ka	кабn		б1	б3	p	q
111	176	0.082	9.102	50.949	183.2261	243.2771	-123.175	60.051	183.2261
222	318.3	0.1	22.2	99.9	333.6089	455.7089	-211.509	122.1	333.6089
444	562.3	0.118	52.392	195.804	595.4162	843.6122	-347.22	248.196	595.4162
666	779	0.14	93.24	286.38	829.9726	1209.593	-450.353	379.62	829.9726
777	865.7	0.147	114.219	331.3905	926.9607	1372.57	-481.351	445.6095	926.9607

Fig 2. p-q graph of failure envelope (via k_a).

According to Table 2 and Figure 2, the gradient of failure envelope is 1.927. Considering the equation of $\tan(x) = \sin\varphi$ and the gradient of graph which is greater than 1, k_a could not be used for analyzing the internal friction angle of direct shear test and converting it to the internal friction angle of triaxial test.

Table 3: The values of p and q (kPa) with k_p as the lateral pressure coefficient

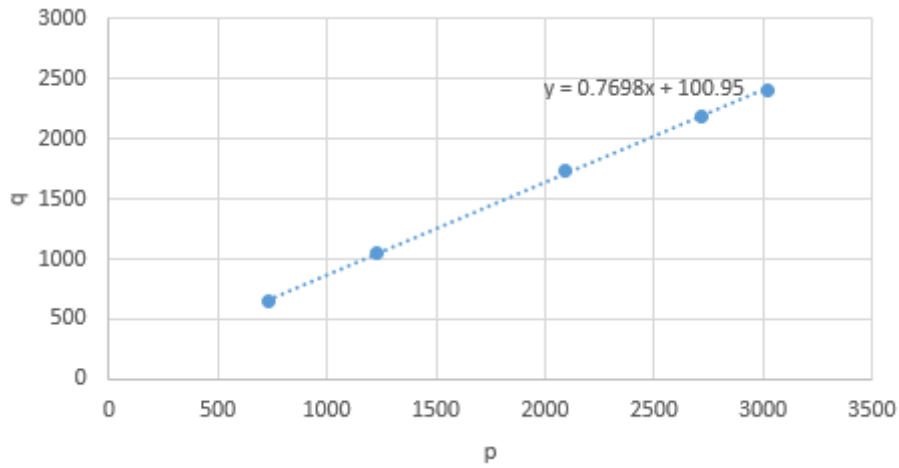| бп	тр	kp	kpбn		б1	б3	p	q
111	176	12.162	1349.982	-619.491	644.0071	1374.498	86.48394	730.491	644.0071
222	318.3	10.06	2233.32	-1005.66	1054.83	2282.49	172.8297	1227.66	1054.83
444	562.3	8.434	3744.696	-1650.35	1743.511	3837.859	350.8372	2094.348	1743.511
666	779	7.153	4763.898	-2048.95	2192.039	4906.988	522.9105	2714.949	2192.039
777	865.7	6.786	5272.722	-2247.86	2408.8	5433.661	616.0614	3024.861	2408.8

Fig 3. p-q graph of failure envelope (via k_p).

Based on Table 3 and Figure 3, the gradient of failure envelope is 0.7698. Considering the equation of $\tan(x) = \sin\varphi$ and the gradient of graph which is less than 1, k_p can be used for analyzing the internal friction angle of direct shear test and converting it to the internal friction angle of triaxial test. It could be concluded that $\sin\varphi=0.7698$ and $\varphi=50.34$. The φ is related to direct shear test, but according to $\varphi_{ps} =1.15\varphi_{tr}$ equation, the internal friction angle of triaxial test is 43.48.

Table 4: The values of p and q (kPa) with $(k_p+ k_p)/2$ as the lateral pressure coefficient

| бп	тр	(ka+kp)/2	(kp+ka)бп/2			б1	б3	p	q
111	176	6.122	679.542	-284.271	334.3441	729.6151	60.92686	395.271	334.3441
222	318.3	5.08	1127.76	-452.88	553.5478	1228.428	121.3322	674.88	553.5478
444	562.3	4.276	1898.544	-727.272	919.2964	2090.568	251.9756	1171.272	919.2964
666	779	3.646	2428.236	-881.118	1176.099	2723.217	371.0185	1547.118	1176.099
777	865.7	3.4665	2693.471	-958.235	1291.376	3026.611	443.8595	1735.235	1291.376
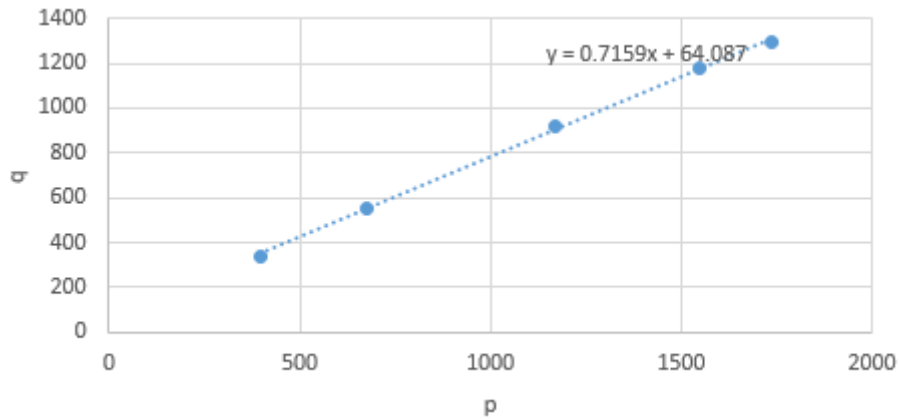
Fig 4. p-q graph of failure envelope (via $(k_p + k_p)/2$).

According to the Table 4 and Figure 4, the gradient of failure envelope is 0.7698. Considering the equation of $\tan(x) = \sin\varphi$ and the gradient of graph which is less than 1, $(k_p + k_p)/2$ can be used for analyzing the internal friction angle of direct shear test and converting it to the internal friction angle of triaxial test. It could be concluded that $\sin\varphi = 0.7159$ and $\varphi = 45.72$. The φ angle is related to direct shear test, but according to $\varphi_{ps} = 1.15\varphi_{tr}$ equation, the internal friction angle of triaxial test is 39.76.

Table 5: The values of p and q (kPa) with $(k_0 + k_p + k_p)/3$ as the lateral pressure coefficient

| бn	тp	(ka+kp+k0	(kp+ka+k0)бn/3		б1	б3	p	q
111	176	4.132	458.652	-173.826	247.3691	532.1951	37.45689	284.826	247.3691
222	318.3	3.447	765.234	-271.617	418.4384	912.0554	75.17861	493.617	418.4384
444	562.3	2.921	1296.924	-426.462	705.7274	1576.189	164.7346	870.462	705.7274
666	779	2.513	1673.658	-503.829	927.7309	2097.56	242.0981	1169.829	927.7309
777	865.7	2.397	1862.469	-542.735	1021.762	2341.496	297.9727	1319.735	1021.762
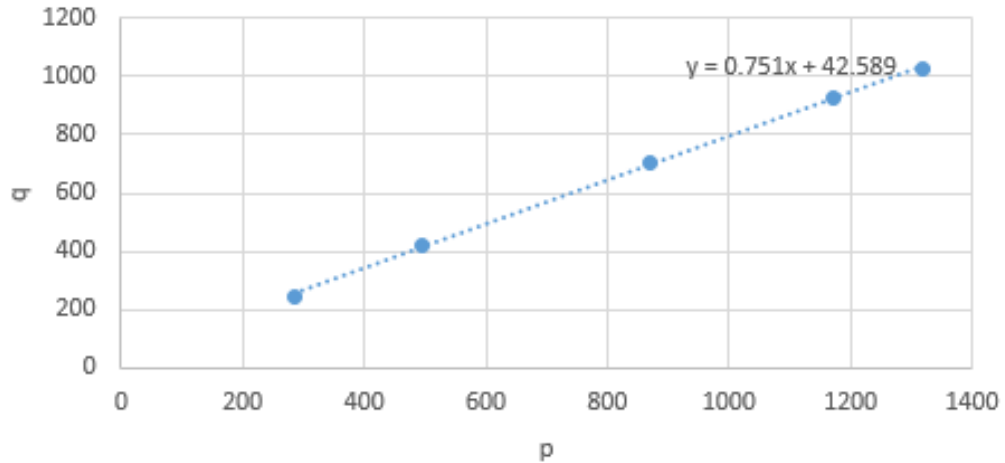
Fig 5. p-q graph of failure envelope (via $(k_0+k_p+ k_p)/3$).

According to the Table 5 and Figure 5, the gradient of failure envelope is 0.751. Considering the equation of $\tan(x) = \sin\varphi$ and the gradient of graph which is less than 1, $(k_0+k_p+ k_p)/3$ could be used for analyzing the internal friction angle of direct shear test and converting it to the internal friction angle of triaxial test. It can be concluded that, $\sin\varphi=0.751$ and $\varphi=48.67$. The φ angle is related to direct shear test, but according to $\varphi_{ps} =1.15\varphi_{tr}$ equation, the internal friction angle of triaxial test is 42.33.
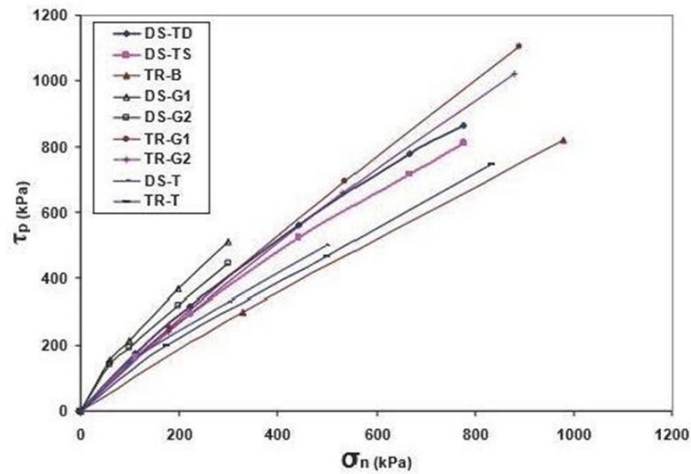

Fig 6. Maximum shear strength vs. vertical surcharge [1].

In Fig. 6, the TR-B line is the failure envelope of large-scale triaxial test. Using the gradient of the corresponding graph, the internal friction angle could be calculated. Moreover, by the Plot Digitizer software, the gradient of 88.68 is shown, thus the internal friction angle of 43.5 is calculated with tan-1 (88.68).

4- Conclusion

In order to study the internal friction angle of the gravel, the stress path and failure envelope of the sample are drawn and investigated in p-q space via different lateral pressure coefficients, ie. at rest (k0), active (ka) and passive (kp) cases, and the combinations of them such as (ka + kp)/2 and (k0 + ka + kp)/3. Then they are compared with triaxial test results. The main conclusions are mentioned below:

● The active (ka) and at rest (k0) lateral pressure coefficients could not be used for analyzing the results of direct shear test.

● lateral pressure coefficients of kp, (ka + kp)/2 and (k0 + ka + kp)/3 can be used for analyzing the results of direct shear test.

●The nearness of the results obtained from the direct shear test in comparison with the triaxial test results would be acceptable if passive lateral pressure coefficient (kp) is used. The internal friction angle of this condition is approximately the same with the internal friction angle of the large-scale triaxial test. In case of using lateral pressure coefficient of (k0 + ka + kp)/3, the results would be less coincident with actual parameters of soil in comparison with the case of using passive lateral pressure coefficient, but the result is still within the expected and appropriate range.

Finally, it could be stated that, using lateral pressure coefficient of (kp + ka) / 2 in direct shear test, the corresponding results are significantly different from the actual parameters of the material. The result is not within the expected and appropriate range.

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